1.

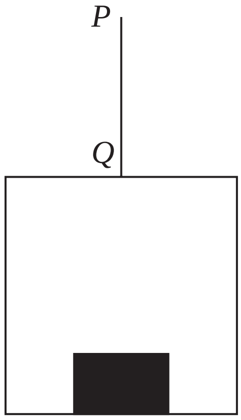


Figure 1

A vertical rope PQ has its end Q attached to the top of a small lift cage.

The lift cage has mass 40 kg and carries a block of mass 10 kg, as shown in Figure 1.

The lift cage is raised vertically by moving the end P of the rope vertically upwards with constant acceleration $0.2\,\mathrm{m\,s^{-2}}$

The rope is modelled as being light and inextensible and air resistance is ignored.

Using the model,

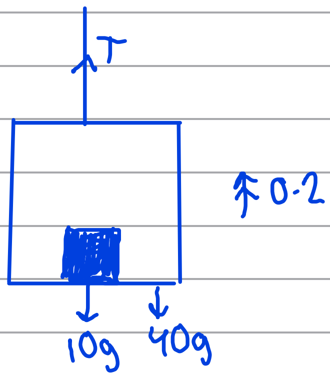
(a) find the tension in the rope PQ

(3)

(b) find the magnitude of the force exerted on the block by the lift cage.

(3)

a)



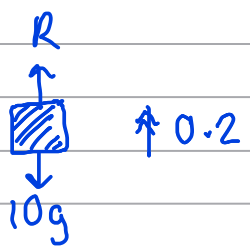
F=ma on whole system:

$R(\uparrow)$: $T - 10g - 40g = 0.2(50)$ ②

$T = 10 + 50 \times 9.8$

$= 500\text{ N}$ ①

b)



R = force exerted on block from lift cage

$R(\uparrow)$: $R - 10g = 0.2(10)$ ②

$R = 2 + 10g$

$= 100\text{ N}$ ①

2.

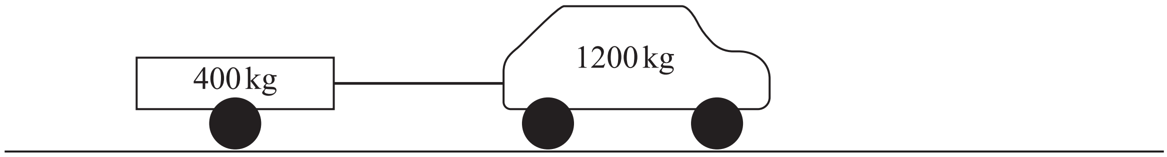


Figure 2

A car of mass 1200 kg is towing a trailer of mass 400 kg along a straight horizontal road using a tow rope, as shown in Figure 2.
The rope is horizontal and parallel to the direction of motion of the car.

- The resistance to motion of the car is modelled as a constant force of magnitude $2R$ newtons
- The resistance to motion of the trailer is modelled as a constant force of magnitude R newtons
- The rope is modelled as being light and inextensible
- The acceleration of the car is modelled as a m s^{-2}

The driving force of the engine of the car is 7400 N and the tension in the tow rope is 2400 N.

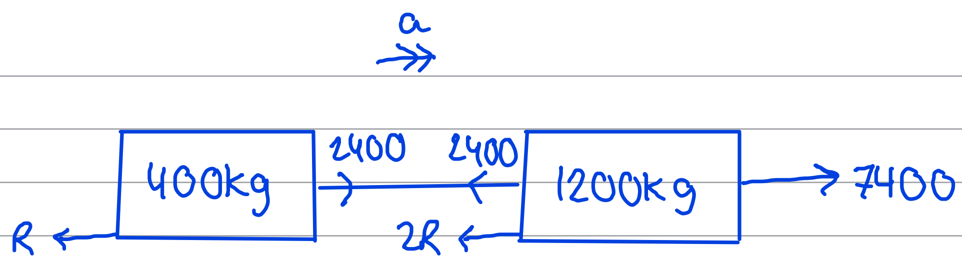
Using the model,
(a) find the value of a
(5)

In a refined model, the rope is modelled as having mass and the acceleration of the car is found to be a_1 m s^{-2}

(b) State how the value of a_1 compares with the value of a
(1)

(c) State one limitation of the model used for the resistance to motion of the car.
(1)

a)



considering car: $R(\rightarrow)$ 1 considering trailer: $R(\rightarrow)$ 1

$7400 - 2R - 2400 = 1200a$ 1 $2400 - R = 400a$ 2 1

$5000 - 2R = 1200a$ 1

solve ① and ② simultaneously using calculator:

$a = 0.5$ ① $R = 2200$

b) a_1 would be less than a ①

c) air resistance will vary depending on speed, so it won't be constant. ①

3.

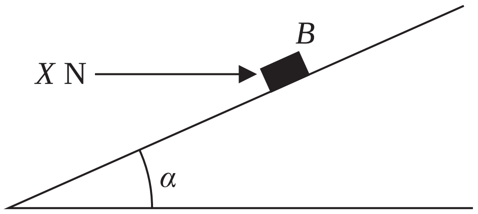


Figure 1

A rough plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$

A small block B of mass 5 kg is held in equilibrium on the plane by a horizontal force of magnitude X newtons, as shown in Figure 1.

The force acts in a vertical plane which contains a line of greatest slope of the inclined plane.

The block B is modelled as a particle.

The magnitude of the normal reaction of the plane on B is 68.6 N.

Using the model,

(a) (i) find the magnitude of the frictional force acting on B,

(3)

(ii) state the direction of the frictional force acting on B.

(1)

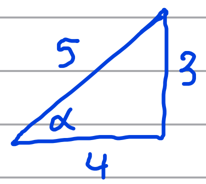
The horizontal force of magnitude X newtons is now removed and B moves down the plane.

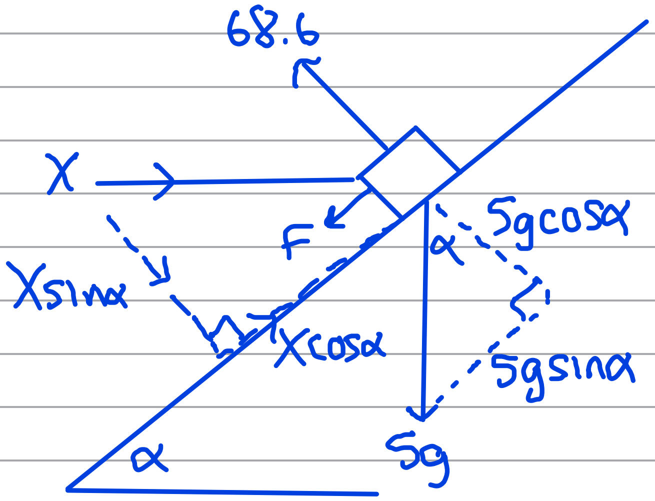
Given that the coefficient of friction between B and the plane is 0.5

(b) find the acceleration of B down the plane.

(6)

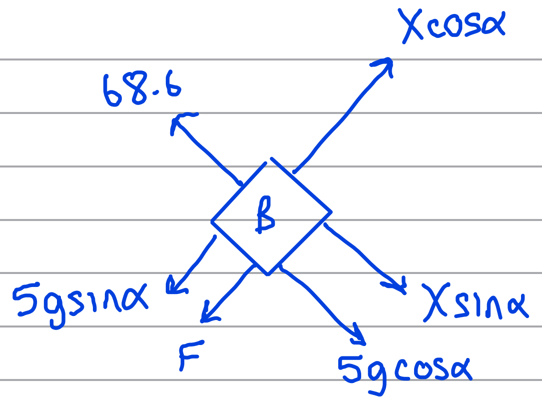
$\tan\alpha = 3/4$





$\sin\alpha = 3/5$

$\cos\alpha = 4/5$



"R( )" means resolve

a)

(i) R(↖): $68.6 = X\sin\alpha + 5g\cos\alpha$ ①

$$\Rightarrow X = \frac{68.6 - 5g\cos\alpha}{\sin\alpha} = 49\text{ N}$$ ①

R(↗): $X\cos\alpha = 5g\sin\alpha + F$

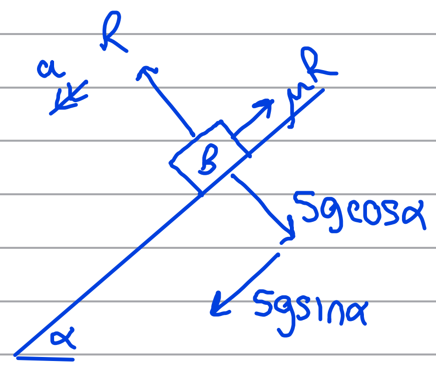
$$\Rightarrow F = X\cos\alpha - 5g\sin\alpha = 9.8\text{ N}$$ ①

(ii) Down the plane ①

Friction opposes motion; without friction the box would slide up the plane, so friction must act down to counteract this.

b) $\mu = 0.5$

$F = \mu R$
$= 0.5R$ ①



- R changes as X is removed
- friction now acts up the plane

R(↖): $R = 5g\cos\alpha = 39.2$ ① ①

R(↙): $5g\sin\alpha - \mu R = 5a$ ① ①

$$\therefore a = \frac{5g\sin\alpha - \mu R}{5}$$

$a = 1.96\text{ ms}^{-2}$ (3sf) ①

4. *[In this question,* **i** *and* **j** *are horizontal unit vectors.]*

A particle P of mass 4 kg is at rest at the point A on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (4\mathbf{i} - \mathbf{j})\,\mathrm{N}$ and $\mathbf{F}_2 = (\lambda\mathbf{i} + \mu\mathbf{j})\,\mathrm{N}$, where λ and μ are constants, are applied to P

Given that P moves in the direction of the vector $(3\mathbf{i} + \mathbf{j})$

(a) show that

$$\lambda - 3\mu + 7 = 0$$

(4)

At time $t = 4$ seconds, P passes through the point B.

Given that $\lambda = 2$

(b) find the length of AB.

(5)

a) $F_1 + F_2 = k\begin{pmatrix}3\\1\end{pmatrix}$

$\begin{pmatrix}4\\-1\end{pmatrix} + \begin{pmatrix}\lambda\\\mu\end{pmatrix} = \begin{pmatrix}3k\\k\end{pmatrix}$ 1

$4 + \lambda = 3k$ ①
$-1 + \mu = k$ ②

sub ② into ①: $4 + \lambda = 3(-1 + \mu)$ 1
$4 + \lambda = -3 + 3\mu$ 1
$\Rightarrow \lambda - 3\mu + 7 = 0$ 1

b) given $\lambda = 2$, so find μ:

$2 - 3\mu + 7 = 0$
$\mu = 3$

$\therefore$ resultant force is

$\begin{pmatrix}4\\-1\end{pmatrix} + \begin{pmatrix}2\\3\end{pmatrix} = \begin{pmatrix}6\\2\end{pmatrix}$ 1

$\underline{F} = m\underline{a}$

$\begin{pmatrix} 6 \\ 2 \end{pmatrix} = 4\underline{a}$ (1)

$\underline{a} = \begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix}$

motion AB:

$\underline{s} = \underline{s}$

$\underline{u} = 0$

$\underline{v} =$

$\underline{a} = 1.5\underline{i} + 0.5\underline{j}$

$t = 4$

$\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2$

$= 0 + \frac{4^2}{2}\begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix}$ (1)

$\underline{s} = \begin{pmatrix} 12 \\ 4 \end{pmatrix}$

u=0 as starts from rest

distance $= |\underline{s}| = \sqrt{12^2 + 4^2}$ (1) $= 4\sqrt{10}$ (1)

5.

Figure 1

A particle P has mass 5 kg.

The particle is pulled along a rough horizontal plane by a horizontal force of magnitude 28 N.

The only resistance to motion is a frictional force of magnitude F newtons, as shown in Figure 1.

(a) Find the magnitude of the normal reaction of the plane on P

(1)

The particle is accelerating along the plane at 1.4 m s^{-2}

(b) Find the value of F

(2)

The coefficient of friction between P and the plane is μ

(c) Find the value of μ, giving your answer to 2 significant figures.

(1)

(a) $R = mg$ reaction = mass × gravity

$= 5 \times 9.8$ (equal to weight, which keeps P on the plane)

$= 49\text{N}$ (1)

(b) $\overrightarrow{\text{force}}$ = mass × acceleration

$28 - F = 5 \times 1.4$ (1) total force $\rightarrow$ = $28 - F$

$28 - F = 7$

$F = 21\text{ N}$ (1)

(c) $\mu = \dfrac{F}{R}$ ← friction, ← normal force

$\mu = 21 \div 49$ (from part a)

$\mu = 0.43$ (1)

6.

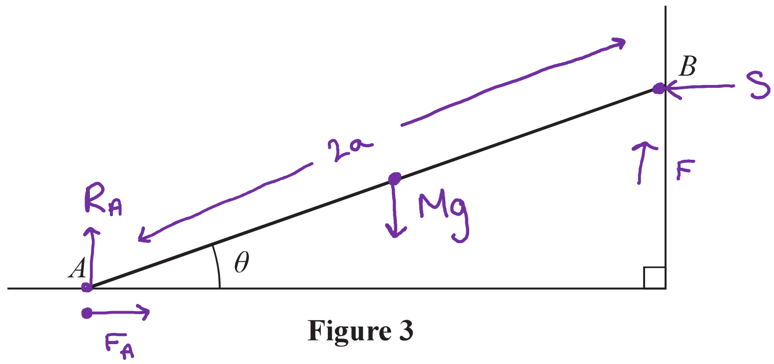


Figure 3

A rod AB has mass M and length $2a$.

The rod has its end A on rough horizontal ground and its end B against a smooth vertical wall.

The rod makes an angle θ with the ground, as shown in Figure 3.

The rod is at rest in limiting equilibrium.

(a) State the direction (left or right on Figure 3 above) of the frictional force acting on the rod at A. **Give a reason for your answer.**

(1)

The magnitude of the normal reaction of the wall on the rod at B is S.

In an initial model, the rod is modelled as being **uniform**.

Use this initial model to answer parts (b), (c) and (d).

(b) By taking moments about A, show that

$$S = \frac{1}{2}Mg\cot\theta$$

(3)

The coefficient of friction between the rod and the ground is μ

Given that $\tan\theta = \frac{3}{4}$

(c) find the value of μ

(5)

(d) find, in terms of M and g, the magnitude of the resultant force acting on the rod at A.

(3)

In a new model, the rod is modelled as being **non-uniform**, with its centre of mass closer to B than it is to A.

A new value for S is calculated using this new model, with $\tan\theta = \frac{3}{4}$

(e) State whether this new value for S is larger, smaller or equal to the value that S would take using the initial model. **Give a reason for your answer.**

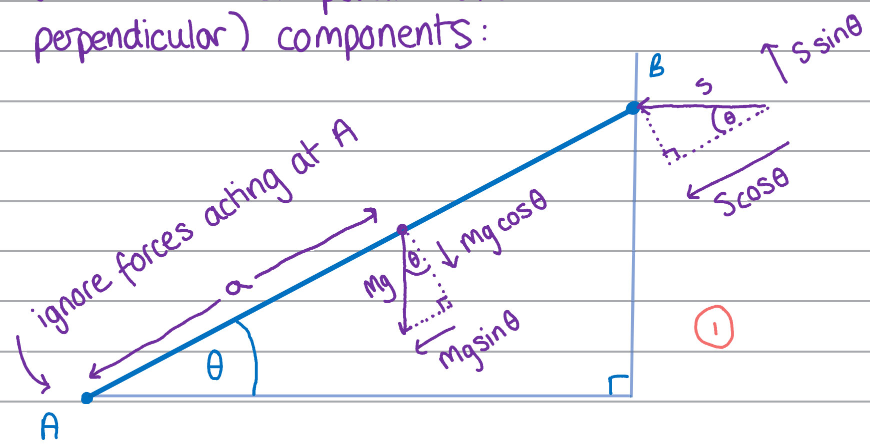
(1)

Question 6 continued

(a) Frictional force at A acts right because it must oppose the normal reaction at B, which acts left. 1

(b) Calculate the horizontal and vertical (or parallel and perpendicular) components:

moment = force × distance from point to force



$$aMg\cos\theta = 2aS\sin\theta$$ 1

÷ 2a

$$\frac{a}{2a}Mg\cos\theta = S\sin\theta$$

÷ sinθ

$$\frac{a}{2a}Mg \times \frac{\cos\theta}{\sin\theta} = S$$

$\cot = \frac{1}{\tan}$, $\tan = \frac{\sin}{\cos}$ ⟹ $\cot = \frac{\cos}{\sin}$

$$\frac{1}{2}Mg \times \cot\theta = S$$ 1

(c) Resolving vertically : $R = mg$ 1

Resolving horizontally: $F = S$ 1

the system is in equilibrium, so vertical and horizontal forces must be equal.

$F = \mu R \Rightarrow \mu R = S \Rightarrow \mu mg = S$ 1

$\frac{1}{2}Mg \times \cot\theta = S$ ← from part (b)

Question 6 continued

$\frac{1}{2}Mg \times \frac{4}{3} = \mu Mg$ (1) ← $\tan\theta = \frac{3}{4} \Rightarrow \frac{1}{\tan\theta} = \frac{4}{3}$

$\div Mg$

$\frac{1}{2} \times \frac{4}{3} = \mu$

$\mu = \frac{2}{3}$ (1)

(d) Forces acting on A: R = normal reaction = Mg

$F = \mu R = \frac{2}{3}Mg$

Magnitude $= \sqrt{F^2 + R^2}$ (1)

$= \sqrt{\left(\frac{2}{3}Mg\right)^2 + (Mg)^2}$ (1)

$= \sqrt{\frac{4}{9}M^2g^2 + M^2g^2}$

$= \sqrt{\frac{13}{9}M^2g^2}$

$= \frac{1}{3}Mg\sqrt{13}$ (1)

(e) New value of S would be larger because the moment of the weight about A would be larger. (1)